

The Sun Asphericities : Astrophysical Relevance

Jean-Pierre ROZELOT¹, Sophie PIREAUX², Sandrine LEFEBVRE³

¹ Observatoire de la Côte d'Azur (OCA)

Département GEMINI - 06130 GRASSE, France

² Observatoire Midi-Pyrénées (OMP, UMR 5562), 31400 Toulouse, France

³ University of California UCLA, Los Angeles, CA 90096-562, USA, on leave from OCA

e-mail : jean-pierre.rozelot@obs-azur.fr

Of all the fundamental parameters of the Sun (diameter, mass, temperature...), the gravitational multipole moments (of degree l and order m) that determine the solar moments of inertia, are still poorly known. However, at the first order ($l=2$), the quadrupole moment is relevant to many astrophysical applications. It indeed contributes to the relativistic perihelion advance of planets, together with the post-Newtonian (PN) parameters; or to the precession of the orbital plane about the Sun polar axis, the latter being unaffected by the purely relativistic PN contribution. Hence, a precise knowledge of the quadrupole moment is necessary for accurate orbit determination, and alternatively, to obtain constraints on the PN parameters. Moreover, the successive gravitational multipole moments have a physical meaning : they describe deviations from a purely spherical mass distribution. Thus, their precise determination gives indications on the solar internal structure. Here, we explain why it is difficult to compute these parameters, how to derive the best values, and how they will be determined in a near future by means of space experiments.

Keywords : sun ; solar rotation ; solar gravitational moments ;
general relativity ; perihelion advances of planets.

1 Introduction

The study of the rotation of stars is not trivial. In theory, the problem is exceedingly simple and can be formulated as follows. Consider a single star that rotates about a fixed direction in space, with an angular velocity Ω . Let us first assume that, for $\Omega = 0$, the star is a gaseous body in gravitational equilibrium. The problem is to determine the outer shape of the star when the initial sphere is set rotating at an angular velocity Ω . Such studies were conducted for the first time by Milne (1923), then fully achieved by Chandrasekhar (1933).

The second point which arises is to ask oneself what will happen if Ω is not constant, not only in latitude (differential rotation) but also throughout the body, from the surface to the core. Today, astronomers are faced with such problems, not only in the solar case, but also for stars. With the advent of sophisticated techniques such as interferometry, one is now able to accurately

determine the geometrical shape of the free boundary of stars, such as Altair or Achernar for which observations of the geometrical envelope have been made by Belle *et al.* (2001), and Domiciano de Souza (2002). But it would be of little or no interest to observe the geometric shape of a star if one would not be able to infer some information on stellar physics. With such an approach, the purpose of theoreticians is to enumerate all the possible angular velocity distributions (from the center to the surface) that are compatible with the observed stellar surface. For stars, Maeder (1999) examined the effects of rotation and wrote the equation of the surface with a rotation law which is differential, but only in the surface layer.

In other words, the knowledge of the angular velocity distribution from the core to the surface, together with the knowledge of the density function (related to the pressure function), completely determine the outer shape of the stars. Different techniques exist to observe such a figure. Once accurately determined, one would be able to go back to the physical properties of the body. This approach is called “*Theory of Figures*”.

The Theory of Figures has been widely used in geophysics and is still used in specific cases, such as for the planet Mars, with an incredible accuracy ($J_2 \text{ Mars} = 1.860718 \cdot 10^{-3}$ according to Yuan *et al.* (2002), from a 75th degree and order model). Since the pioneering work of Clairaut (1743), who was the first to compute the flattening of a rotating body, and Bruns (1878) who introduced the concept of “Figure of the Earth”, considerable work has been done, mainly by Radau (1885), Wavre (1932), Molodensky (1988), Moritz (1990), and many others since then. The basic principle of this theory is to determine the outer surface of a rotating body, assuming a knowledge of the mass and angular velocity distributions. In the case of the Earth, the main emphasis was on the determination of a global ellipsoid. The theory of hydrostatic equilibrium was considered as the best means for determining the terrestrial flattening f , until the advent of artificial satellites in 1957. Today, considerable accuracy is reached that allows the inverse problem to be solved, that is to say to determine anomalies inside the Earth mantle. The Earth external gravitational potential is developed in a series of so-called spherical harmonics of degree l and order m . The dimensionless coefficients represent the different distortions from a pure sphere. Thanks to artificial satellites, they are now computed up to a very high degree ($J_2 \text{ Earth} = 4.84165209 \cdot 10^{-4}$ according to the EGM96 tide free model adopted by the International Earth Rotation Service in (McCarthy, 2003), from a 360th degree and order model), and substantial progresses are made in the knowledge of our planet.

Curiously, for the Sun, the situation is hardly that of the Earth which prevailed before 1957. The flattening of the Sun is still poorly known and we hardly know if there is a variability linked to the solar cycle. The Theory of Figures was not developed for the Sun until Rozelot *et al.* (2001). The difference between the Sun and the Earth lies in the fact that, in the latter case, the rotation can be considered as solid and the outer shape is mainly due to strong density variations inside the body, on which are superimposed tides, oceanic currents and atmospheric effects. On the contrary, in the solar case, the density can be

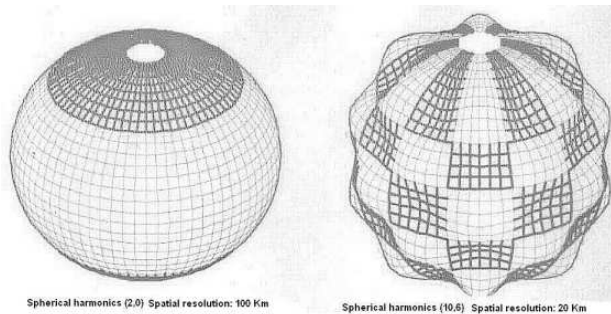


FIG. 1 – Left : 3d sketch of the ($l= 2, m= 0$) mode; the Sun is nearly a spheroid. Right : 3d sketch of the ($l= 10, m= 6$) mode; distortions from a spherical surface appear, on a very exaggerated scale.

considered as nearly homogeneous on successive shells, whereas the rotation is strongly dependent on the latitude. Solar modeling is further complicated by the fact that this differential rotation is not only a surface phenomena, as can be seen for example by observing spots or faculae, but is anchored in depth, at least down to 0.7 solar radii. The computation of the solar gravitational potential may be not too difficult in its formalism, and is nearly the same as for the Earth itself. However the computation of the centrifugal potential is very complicated and it cannot be reduced to computation of potentials on successive cylinders (or thin zonal rings) in which the rotation is taken as uniform. Moreover, we have shown that the centrifugal force corresponding to the commonly adopted differential rotation law (a development in cosine of the colatitude) does not derive from a potential. Hence, we proposed an alternative rotation law (Rozelot and Lefebvre, 2003). The complexity of the rotation profile (see for example Eq. 10) indicates that the photospheric shape is highly sensitive to the interior structure. Thus, in principle, accurate measurements of the limb shape distortions, which we call **“asphericities”** (i.e. departures from the “heloid”, the reference equilibrium surface of the Sun), combined with an accurate determination of the solar rotation provides useful constraints on the internal layers of the Sun (density, shear zones, surface circulation of the plasma...). Fig. 1 shows such asphericities that can be seen at a given spatial resolution. Alternatively, theoretical upperbounds could be inferred for the flattening which may exclude incorrect/biased observations.

Thus, even if we know that the Solar Theory of Figures is limited due to truncation errors, the learning is rich.

In this paper, we will successively focus on : (i) the definition of the solar gravitational moments, (ii) how to compute these moments, (iii) their relevance to astrophysics and (iv) how to measure such quantities. To conclude, we will show why space missions are needed.

2 Definition of the solar gravitational multipole moments

For an axially symmetric distribution of rotating matter, the outer gravitational field can be expressed as :

$$\phi_{grav\ out}(r, \theta) = -\frac{GM_{\odot}}{r} \left[1 - \sum_{n=1}^{\infty} \left(\frac{R_{\odot}}{r} \right)^{2n} J_{2n} P_{2n}(\cos \theta) \right] \quad (1)$$

where G is the gravitational constant ; M_{\odot} and R_{\odot} , the solar mass and radius ; J_{2n} are the gravitational multiple moments ; P_{2n} , the Legendre polynomials ; r and θ , respectively the distance from the Sun centre and the angle to the symmetry axis (colatitude).

The first terms are called

- for $n = 1$, J_2 the solar quadrupole moment
- for $n = 2$, J_4 the solar octopole moment
- for $n = 3$, J_6 the solar dodecapole moment
- ...

For the Sun, as we will see later, terms of higher degree are nearly null. The J_n coefficients are dimensionless quantities providing information on how the mass and velocity distributions act inside the Sun to finally render the outer visible shape non spherical. The amplitude, peak to peak, of such asphericities does not exceed some 20 milliarcseconds (abbreviated as “mas”), as shown by the theory itself (Lefebvre and Rozelot, 2004) or by observations (Rozelot *et al.*, 2003).

3 Computation of the solar gravitational moments

The starting point is to consider the equation of motion for an ideal fluid

$$\rho \vec{x} = \rho \vec{g} - \vec{\nabla} p \quad (2)$$

where \vec{x} is the acceleration of the fluid ; ρ is the density ; \vec{g} , the gravitational force ; and $\vec{\nabla} p$, the pressure gradient. For hydrostatic equilibrium, there is no motion, hence $\vec{x} = 0$. For a nonrotating body, the force acting on particles is given by $\vec{g} = -\vec{\nabla} \phi_{grav}$, where ϕ_{grav} is the gravitational potential. Hence Eq. 2 reduces to ¹

$$0 = -\rho \vec{\nabla} \phi_{grav} - \vec{\nabla} p \quad (3)$$

When the body is rotating, the force acting on particles is the gradient of the gravity potential ($\phi_{total} = \phi_{grav} + \phi_{rotation}$) ; that is, we must add the centrifugal

¹The curl of this equation is $\vec{\nabla} \rho \times \vec{\nabla} \phi = 0$, so that the normals to surfaces of constant ρ and ϕ point in the same direction. As a consequence, the surfaces of constant ρ and ϕ coincide. Using the perfect gas equation of state (with constant chemical composition), it can be shown that surfaces of constant ρ , ϕ , T and P all coincide. This is known as the Von Zeipel theorem : any internal source of distortion in the gravitational field at the surface will manifest itself as a change of shape in the solar surface layer. Thus, measuring the shape of the surface layers is equivalent to measuring surfaces of constant gravitational potential.

force, so that Eq. 3 becomes

$$\vec{\nabla} p = -\rho \vec{\nabla} \phi_{grav} + \Omega^2(r, \theta) \vec{s} \quad (4)$$

where $\Omega(r, \theta)$ is the angular velocity and \vec{s} a vector perpendicular to the rotation axis directed outwards.

There are two ways to compute the coefficients J_n .

- The first method solves Eq. 4 by considering that asphericities are small quantities : the solution is the sum of a spherically symmetric potential (indexed by a subscript 0) and a perturbation (nonspherically symmetric, indexed by a subscript 1) :

$$\phi_{grav}(r, \theta) = \phi_{grav\ 0}(r) + \phi_{grav\ 1}(r, \theta) \quad (5)$$

Such an approach was taken by Goldreich and Schubert (1968), Ulrich and Hawkings (1981), Pijpers (1998), Armstrong and Kuhn (1999), Godier and Rozelot (1999) or Roxburgh (2001). Differences lie in the way Eq. 4 is projected on appropriate coordinates. Except for Armstrong and Kuhn who used a series of vector spherical harmonics to avoid truncation errors, all the above mentioned authors made a projection on Legendre polynomials $P_{2n}(\cos\theta)$, so that

$$\phi_{grav\ 1}(r, \theta) = \sum_{n=1}^{\infty} \phi_{grav\ 1,2n}(r) P_{2n}(\cos\theta)$$

According to this, Eq. 2, supplemented with Poisson's equation

$$\nabla^2 \phi_{grav} = 4\pi G \rho \quad (6)$$

and boundary conditions at $r = 0$, where $\phi_{grav} = 0$ or at the surface $r = R_{\odot}$ where $\phi_{grav} = \phi_{grav\ out}$, finally lead to

$$J_{2n} = \frac{R_{\odot}}{GM_{\odot}} \phi_{grav\ 1,2n}(R_{\odot}) \quad (7)$$

The function $\phi_{grav\ 1,2n}$ is the solution to a differential equation requiring the knowledge of $\rho(r)$ and $\Omega(r, \theta)$. Different models can be used for the density or the rotation law. In general $\rho(r)$ is taken either from Richard (1999), Christensen-Dalsgaard *et al.* (1996) or Morel (1997). The rotation law can be either the standard rotation law (in cosine, see Eq. 8) or can be derived from the analytical law of Kosovichev (1998) or Corbard *et al.* (2002) (see Eq. ??). Fig. 2 -left- shows solar rotation profiles, from the surface down to $0.3 R_{\odot}$, obtained by a 1.5 dimensional inversion of data from the *MDI* (Michelson Doppler Imager) experiment on board the SOHO mission (Di Mauro, 2003). Fig. 2 -right- shows the Corbard rotation model deduced from observations of the *MDI* *f*-modes between May 1996 and April 2001 (Dikpati *et al.*, 2002).

A complete expression of $\phi_{grav\ 1,2}$ and $\phi_{grav\ 1,4}$ was provided by Armstrong and Kuhn (1999), using the standard rotation law. However, surface plasma observations allow to constrain analytical rotation models. The first attempt to derive

an analytical rotation law from helioseismic data has been made by Kosovichev (1996). Using this law, Godier and Rozelot (1999), then Roxburgh (2001) computed the gravitational moments (see Table 1). The discrepancy between the values obtained can be explained by the use of different density models (the model of density used here is that of Richard, 1999) and by the way the differential equation Eq. 7) is integrated. Kosovichev noted that a subsurface shear layer results when the helioseismically obtained internal rotation is matched with the surface rotation. Hence, we suspect that the subsurface rotation rate, in this very thin layer (called the *leptocline*), may play an important role. To check this point, we used the rotation model described in (Dikpati *et al.*, 2002). This model presents the following characteristics : (i) the rotation rate is constant in the radiative interior (core), i.e. $\Omega_o(r, \theta) = 435 \text{ nHz}$ as in Kosovichev (1996); (ii) the location and width of the tachocline are assumed to be independent from latitude; (iii) the rotation rate at the top of the tachocline and at the surface are given by

$$\Omega(r_{cz}, \theta) = \Omega_{eq}(r) + a_2 \cos^2 \theta + a_4 \cos^4 \theta \quad (8)$$

with $a_2 = -61 \text{ nHz}$ and $a_4 = -73.5 \text{ nHz}$; (iv) the radial rotation gradient near the surface is assumed to be constant at a given latitude and its latitudinal dependence is given by $\beta(\theta)$, which is a polynomial function of $\cos \theta$: $\beta(\theta) = \beta_0 + \beta_3(\cos \theta)^3 + \beta_6(\cos \theta)^6$. The transition between layers (of constant $\partial\Omega/\partial r$ at a given latitude) is constructed using error functions, i.e. $\Xi_x(r) = 0.5 \{1 + \text{erf}[2(r - r_x)/w_x]\}$ where w is the width of the transition, and x stands either for the tachocline (tac), the convective zone (cz) or the surface (s). The final expression of the solar internal rotation law is given by Eqs. 1, 2 and 3 of Dikpati *et al.*, (2002) or, alternatively, Eqs. 1, 2, 4 and 5 of (Corbard *et al.*, 2002). Values of the parameters Ω_o , a_2 and a_4 lead to $\Omega_{cz} = 453.5 \text{ nHz}$ (but other values of a_2 and a_4 are found by other authors -see Lefebvre and Rozelot, 2004); we adopted $w_{tac} = w_{cz} = w_s = 0.05 R_\odot$, $r_{tac} = 0.69 R_\odot$, $r_{cz} = 0.71 R_\odot$ and $r_s = 0.97 R_\odot$. Parameter β_0 has been taken as $437 \text{ nHz}/R_\odot$, instead of 891.5 used by Kosovichev(1996), and we used $\beta_3 = -214$ and $\beta_6 = -503 (\text{nHz}/R_\odot)^2$. The main advantage of this formalism resides in a modeling of the radial differential rotation in the subsurface. The order of magnitude of this outward gradient is $-5.7 \cdot 10^{-16} \text{ m}^{-1} \text{ s}^{-1}$ at the equator. In a shell with a thickness of about $0.03 R_\odot$, the radial rotation gradient $\partial\Omega/\partial r$ is negative from the equator to about 50° of latitude, crosses zero around 50° and is positive beyond this latitude. Basu *et al.* (1999) deduced a similar behaviour from their analysis of the splitting of high-degree f -modes, but finding a reversal of the radial gradient in a zone above $0.994 R_\odot$ only.

After solving equation Eq. 7, it turns out that

$$J_2 = 2.28 \cdot 10^{-7}$$

This computed value is very sensitive to the surface parameters (a_2 and a_4 , see Eq. 8) (latitudinal rotation gradient), and to the latitudinal differential rotation

²Dikpati *et al.* (2002) alternatively considered $\beta_3 = 0$ and $\beta_6 = -1445 (\text{nHz}/R_\odot)$, but computations do not show strong differences in J_2 , only 3 per 1000.

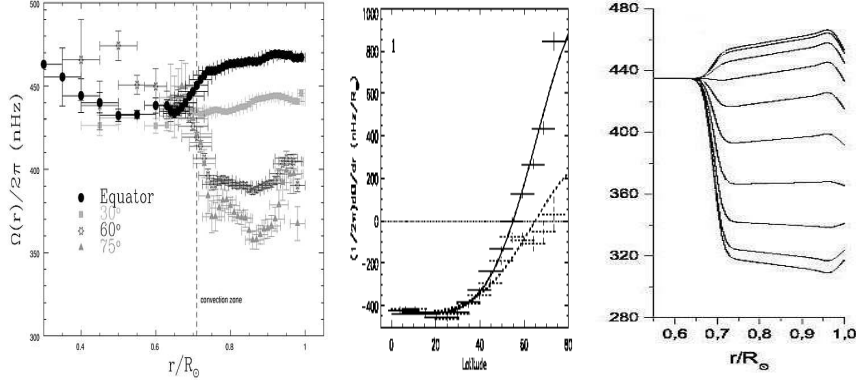


FIG. 2 – Left : Solar rotation profiles from $0.3 R_{\odot}$ to the surface for different latitudes, from the Equator (top curve) to the Pole (bottom curve), obtained by a 1.5 dimensional inversion of SOHO-*MDI* data. Courtesy of Maria di Mauro, 2003. Center : Radial gradient of the angular velocity as a function of latitude. See (Corbard *et al.*, 2002) for further details. Right : Solar rotation profiles from $0.55 R_{\odot}$ to the surface for different latitudes, computed every 10° from the Equator (top curve) to the Pole (bottom curve), with the analytical rotation model of Corbard *et al.* (2002). This model takes into account the sub-surface gradient, as shown in the center plot (dotted line).

in the convective zone. It can reach a maximum of $2.62 \cdot 10^{-7}$ and a minimum of $1.80 \cdot 10^{-7}$, that is to say a 15% variation. The situation is worse for J_4 where the excursion of the computed values is more than 25%. We conclude that (i) helioseismic rotation rates lead to underestimate values of J_n by comparison to values deduced from the theory of Figures (see next section and Table 1) and (ii) the multipole moments (J_2 and J_4), seem to be sensitive to the physical mechanisms which act at the near surface. Mecheri *et al.* (2004), using the same formalism but a different model of density, reached nearly the same conclusions.

- The second method consists in writing the radius r as a function of the colatitude θ ,

$$r = R_{sp} \left[1 + \sum_{n=1}^{\infty} c_{2n} P_{2n}(\cos\theta) \right] \quad (9)$$

where R_{sp} is the radius of the best fitting sphere passing through the poles and the equator of the Sun (for an ellipsoid of revolution with an equatorial radius a and a polar radius b , $R_{sp} = (a^2b)^{1/3}$). It is only a matter of algebra to substitute the above development (Eq. 9) in the external gravitational potential (Eq. 1). It is a bit more complicated to determine a centrifugal force in Eq. 4 corresponding to the rotation law in (r, θ) and which derives from a potential. Adopting the

following rotation law fulfills this requirement :

$$\Omega(r_p) = \Omega_{pol} \left[1 + \sum_{i=1}^{\infty} b_{2i} r_p^{2i} \sin^{2i} \theta \right]^{1/2} \quad (10)$$

where $r_p = r/R_{sp}$ is the normalized radial distance from the solar center at a given depth and Ω_{pol} is the angular velocity at the pole. Using the solar Greenwich data base, Javaraiah and Rozelot (2002) fitted the parameters :

$$b_2 = +0.442 \text{ and } b_4 = +0.056 \text{ at the surface } (r_p = 1); \quad \Omega_{pol} = 381.8 \text{ nHz}$$

The resulting centrifugal potential can then be expressed on the basis of Legendre polynomials and the total potential is written as $\phi_{total} = \Omega_{pol}^2 R_{sp}^2 h(f, J_n)$ where f is the flattening of the Sun ($f = (a - b)/a$ with the above definitions). To determine the J_n , it is sufficient to consider ϕ_{total} as independent from θ , that is, the coefficients of P_{2n} vanish. The fact that the analytical differential rotation models lead to a significantly lower value of the flatness in comparison to a uniform rotation model can only be interpreted in terms of a positive outward rotation gradient in the subsurface.

A full description of the method can be found in Rozelot and Lefebvre (2003) or Lefebvre (2003).

The two methods are slightly different. The first one provides the coefficients J_n if the mass distribution **and** rotation rate are known. It does not yield a determination of the flattening f , and there is no means to link f with J_n without strong assumptions. By contrast, the second method requires an analytical rotation law (deriving from a potential), and it leads to values of J_n expanded in powers of f , together with the determination of the successive c_n .

4 Results

According to the rotation law adopted, the following results are obtained.

1. For uniform (rigid) rotation, Chandrasekhar's computations yield a surface flatness f that depends on the rotation rate :

$$f = (0.5 + 0.856 \rho_m / \rho_c) \alpha \quad (11)$$

where $\alpha = \Omega^2 R_{\odot} / g$. The ratio of central to mean density is ρ_c / ρ_m and g is the surface gravity. For the Sun, one adopts the following values : $\Omega = 461.6 \text{ nHz}$, $g = 2.7 \cdot 10^4 \text{ cm/sec}^2$, $R_{\odot} = 6.96 \cdot 10^{10} \text{ cm}$, $\rho_m = 1.409 \text{ g/cm}^3$, $\rho_c = 90 \text{ g/cm}^3$. Accordingly, we find $\alpha = 2.17 \cdot 10^{-5}$ and $f = 1.11 \cdot 10^{-5}$.

2. For differential rotation :

- (a) The flatness is decreased from $1.11 \cdot 10^{-5}$ to about $8.85 \cdot 10^{-6}$, in contradiction with the theoretical work of Maeder (1999). This can be

explained only if $\partial\Omega/\partial r$ is > 0 , a behaviour observed at latitudes greater than 50° (Corbard, 2000; Basu *et al.*, 1999). It is not yet clearly established whether this flatness is time-dependent or not.

- (b) Different values of the successive gravitational moments, theoretical or deduced from observations, are given in Table 1.

It can be seen that the first method, using helioseismic data, leads to multipole moment values lower than those obtained with the second method. However, the octopole moment, J_4 , is much more sensitive than the quadrupole moment, J_2 , to the presence of latitudinal and radial rotations in the convective zone. Taking into account recent values of the subsurface rotational gradient, as reported by Corbard and Thompson (2002), the two moments J_2 and J_4 are extremely sensitive to physical mechanisms acting just below the surface. This point seems to confirm the existence of a very thin transition layer, as we already suspected, that we call the *leptocline* (Godier and Rozelot, 2001).

5 Relevance of precise solar gravitational moments

The determination of the outer shape of the Sun has at least four major astrophysical consequences :

1. *Accurate determination of the successive gravitational multipole moments*, and J_2 in particular. Until recently, J_2 was not really known, values ranging from $1.0 \cdot 10^{-7}$ (Ulrich and Hawkins, 1981) to $36 \cdot 10^{-7}$ (Gough, 1982). However, on one hand, it is expected that the flattening f be lower than Roche's limit, which is, in the solar case $2.7 \cdot 10^{-5}$. This means that J_2 must be lower than $1.18 \cdot 10^{-5}$. On another hand, we were able to fix an upper bound to the quadrupolar moment, $J_{2 \max} = 3.0 \cdot 10^{-6}$, meaning f is lower than $1.38 \cdot 10^{-5}$ (Rozelot and Bois, 1998). Beyond this value, there would be observational incompatibilities for specific astrophysical phenomena such as lunar librations. Helioseismic data always lead to lower values of the calculated parameters. That is the case for instance for the photospheric radius ($6.95508 \cdot 10^{10} \text{ cm}$ instead of $6.9599 \cdot 10^{10} \text{ cm}$), for the flattening itself ($6 \cdot 10^{-6}$ instead of $9 \cdot 10^{-6}$), for the velocity rate at the equator (452.3 nHz instead of 467.3 nHz) (Javaraiah, 1995³, and also for J_2 . In this context, Pireaux and Rozelot (2003) adopted the following theoretical range for the solar quadrupole moment, based on helioseismic data (Paternò *et al.*, 1996; Pijpers, 1998) and on a solar stratified model taking into account latitudinal differential rotation (Godier and Rozelot, 1999),

$$J_2 = (2.0 \pm 0.4) \cdot 10^{-7} \quad (12)$$

whereas the theory of Figures (Lefebvre, 2003) yields

$$J_2 = (6.52 \pm 2.55) \cdot 10^{-7} \quad (13)$$

³solar rotation rate deduced from the sunspots is greater than 461 nHz, see for instance Table 3.2 in Lefebvre, 2003.

These discrepancies still need to be resolved.

2. Accurate determination of the perihelion advance of a planet, for which a knowledge of the solar quadrupole moment would help to determine the relativistic post-Newtonian parameters γ and β . Indeed, the perihelion advance (with respect to the classical Keplerian prediction) of planets (ΔW) is a combination of a purely relativistic effect and a contribution from the Solar quadrupole moment. It is given by the following expression⁴ :

$$\Delta W = \Delta W_{0,GR} \left[\frac{1}{3}(2\alpha^2 + 2\alpha\gamma - \beta) - F(\alpha)J_2 \right] \quad (14)$$

where $\Delta W_{0,GR}$ represents the contribution due to General Relativity with J_2 set to zero ; α , β , γ , the parameters describing the theory of the gravitation in the parameterized post-Newtonian formalism ; $\mathcal{F}(\alpha)$ is a coefficient depending on the considered planet (see Pireaux and Rozelot, 2003) for further developments. In General Relativity, $\alpha = \beta = \gamma = 1$, so that the perihelion advance of planets can be written as : $\Delta W = 42.981 [1 - \mathcal{F} J_2]$ (arcsec/cy), with $\mathcal{F} = [R_\odot^2 c^2 (3 \sin^2 i - 1)] / [2GM_\odot a(1 - e^2)]$ where G , c , R_\odot and M_\odot have their usual meaning ; and a , e and i are respectively the semi-major axis, the eccentricity and the inclination of the orbit of the body in question. The contribution of J_2 to ΔW is relevant. In the case of Mercury, $\mathcal{F} = +2.8218 \cdot 10^3$.

The parameter γ is constrained by the CASSINI doppler experiment (Bertotti *et al.*, 2003) to

$$\gamma - 1 = (-2.1 \pm 2.3) \cdot 10^{-5} \quad (15)$$

while observational constraints on the Nordtvedt effect (Williams *et al.*, 2001) , combined with the observational range for γ , leads to

$$\beta \in [0.9998; 1.0003] \quad (16)$$

Hence, it turns out that General Relativity is not excluded by those Solar System experiments. However, General Relativity would be incompatible with the Mercury perihelion advance test if $J_2 = 0$ was assumed. But with the adopted theoretical range for J_2 in Eq. 12, General Relativity agrees with this latter test... and there is still room for an alternative theory too (see Fig. 3).

3. Accurate determination of the precession of the orbital plane of a planet about the Sun's polar axis. J_2 also contributes to this relativistic effect, which is unaffected by the relativistic PN combination $(2\alpha^2 + 2\alpha\gamma - \beta)$. The precession of the orbital plane is more easily discernible for moderately large values of the inclination i .

4. Modern planetary ephemerides now include a non null value of J_2 . However, they are presently not able to infer simultaneously all the usual parameters (masses, radii,..., PN parameters) and J_2 from fits to observational data, due to strong correlations. The order of magnitude adopted for J_2 corresponds to

⁴In some references, the coefficient of the term containing the contribution of the orbital inclination is improperly written.

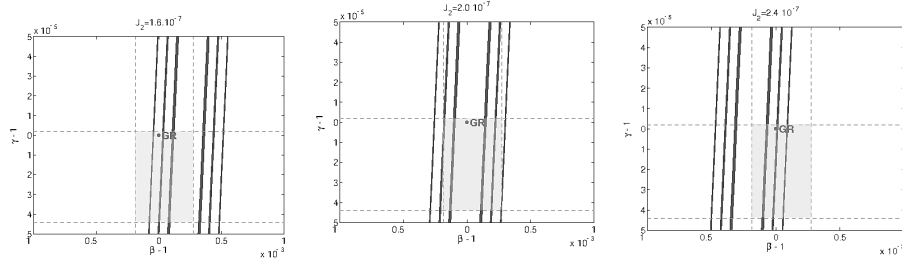


FIG. 3 – For a given value of J_2 the perihelion advance of Mercury constitutes a test of the PN parameters β and γ .

In the β (Eq. 16) and γ (Eq. 15) plane (α set to 1), we have plotted 1σ (the smallest), 2σ and 3σ (the largest) confidence level ellipses. Those are based on the values for the observed perihelion advance of Mercury, ΔW_{obs} , given in the literature and summarized in reference (Pireaux and Rozelot, 2003). Remark also that the position of the ellipses varies according to the value of J_2 chosen; but, their orientation is determined by the combination $(2\alpha^2 + 2\alpha\gamma - \beta)$ that appears in the expression for ΔW . Nevertheless, General Relativity is still in the 3σ contours for the allowed theoretical values of J_2 argued by the authors (Eq. 12). Fig. 1 a, b, c represent the confidence contours for β , and γ , J_2 fixed to its minimum, average and maximum value respectively (Eq. 12).

10^{-7} , but, for example, the estimation of the PN parameters is rather tolerant to the assumed value of J_2 .

6 Observations

Measurements of J_n are not directly accessible.

The multipole moments could be measured dynamically by sending and accurately tracking a probe carrying a drag-free guidance system to within a few radii of the solar center. The J_n are then inferred from the precise determination of the trajectory (Pireaux and Rozelot, 2003).

Alternatively, the J_n can be inferred from in orbit measurement of solar properties (Rozelot and Godier, 2002). Indeed, solar asphericities, encoded mainly by the first two coefficients c_2 and c_4 (Eq. 9) can be observed. An estimate of these two coefficients has been recently derived from SOHO-*MDI* space-based observations (Armstrong and Kuhn, 1999) :

$$c_2 = (-5.27 \pm 0.38) 10^{-6} \text{ and } c_4 = (+1.3 \pm 0.51) 10^{-6}.$$

These results were obtained by measuring small displacements of the solar-limb darkening function (for further details, see Kuhn *et al.*, 1998), and the c_n coefficients are comparable to an isodensity surface level (see footnote 1). >From Earth-based observations at the scanning heliometer of the Pic du Midi Observatory, we also obtained estimates of c_2 and c_4 coefficients. We have shown that asphericities can be observed. A bulge appears to extend from the equator to the edge of the royal zones (about 50° latitude), with a depression beyond (Rozelot

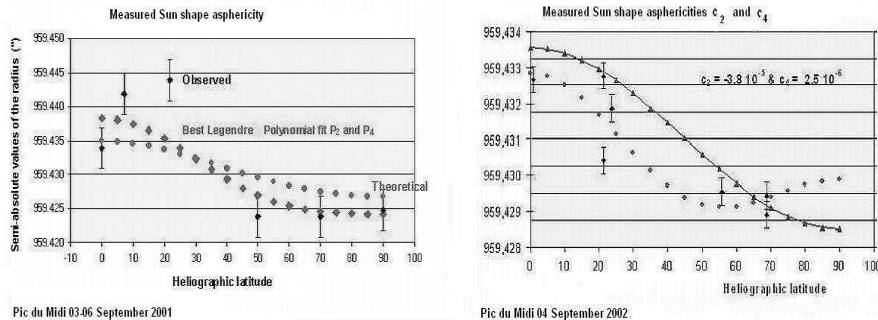


FIG. 4 – Observations of the solar asphericities by means of the scanning heliometer at the Pic du Midi Observatory, in 2001 (left) and in 2002 (right). On the left plot, c_2 and c_4 derive directly from Eq. 9, whereas, on the right plot, these c_n come from a fit to the best ellipsoid passing through the measured points.

et al., 2003). Such a distorted shape can be interpreted through the combination of the quadrupole and hexadecapole terms, which, as shown previously, directly reflect the non uniform velocity rate in surface. Moreover, this distribution implies a thermal wind effect, blowing in the low density surface layers of the Sun, from the poles towards the equator, just as in terrestrial meteorology. This phenomenon was recently studied by Lefebvre and Rozelot (2004) who explained for the first time the existence of small distortions, not exceeding some 20 mas of amplitude. Fig. 4 illustrates those observations. Accordingly, We found :

$$\begin{aligned} \text{Year 2000 : } c_2 &= (-7.6 \pm 0.2) 10^{-6} \text{ and } c_4 = +2.2 10^{-6} \\ \text{Year 2001 : } c_2 &= (-1.1 \pm 0.5) 10^{-5} \text{ and } c_4 = +3.4 10^{-6} \\ \text{Year 2002 : } c_2 &= (-3.8 \pm 0.8) 10^{-5} \text{ and } c_4 = +2.5 10^{-6} \end{aligned}$$

The differences in the estimates stem from the difficulty of observations, mainly due to seeing conditions. The mean observed value of c_2 , $-7.5 10^{-6}$, is not too far from the theoretical one, $\approx -(2/3)f \approx -5.9 10^{-6}$ with $f = 8.9 10^{-6}$ based on a solar model with a differential rotation law. The coefficient c_4 remains difficult to match with the theory, which predicts $+(12/35)f^2$ for a uniform rotation law. The only explanation is that the distorted shape coefficient c_4 is very sensitive to surface phenomena.

7 Conclusions

The main conclusions that can be drawn from this review are the following :

1. Helioseismic data, even when reliable, imply a solar surface rotation rate in conflict with other observed photospheric rotation data. As a consequence, the values of both the solar gravitational moments (J_n) and the shape coefficients (c_n) are slightly different, larger in the latter case. These discrepancies still need to be resolved.
2. The latitudinal dependence conveys sub-surface physical mechanisms that

can be explained theoretically. Thus, in spite of the fact that ground-based observations are altered by seeing effects amplifying or superimposing noise, it could be suggested that the solar shape is not a pure spheroid. It is clear that improvements on this question will depend on better understanding of the sub-surface layers and on better measurements of the limb shape.

3. The exact shape of the Sun critically depends on the rotation law in the external layers.

4. In the setting of General Relativity, it is clear that accurate measurements of the perihelion advance of small planets such as Icarus will help to better determine the quadrupole moment (dynamical estimate). Measurements of the multipole moments (J_n) will be obtained as a by-product of two distinct space missions : Beppi-Colombo (scheduled for 2009) and GAIA (scheduled for 2010). These missions are essential for future developments on that question.

5. In-orbit solar measurements with sufficient resolution are also essential. Measurements of the asphericities (c_n) and determination of the true helioid are one of the main goals of the future space mission PICARD, scheduled to be launched by 2008 ; and also the main targets of balloon flights, such as SDS flights.

Acknowledgements. We thank Thierry Corbard for reviewing the manuscript and his helpful comments.

Références

- [1] Armstrong, J. and Kuhn, J. R. : 1999, *Astrophys. Jour.*, **525**, 533.
- [2] Basu, S., Antia, H.M. and Tripathy, S.C. : 1999, *Astrophys. Jour.*, **512**, 458.
- [3] Belle Van, G.T., Ciardi, D.R., Thompson, R.R., Akeson, R.L. and Lada, E. : 2001, *Astrophys. Jour.*, **559**, 1155.
- [4] Bertotti, B., Iess, L. and Tortora P. : 2003, *Nature*, **425**, 374.
- [5] Bois, E. and Girard, J-F. : 1999, *Celest. Mec. & Dyn. Astron.*, **73**, 329 -338.
- [6] Bruns, H. : 1878, “Die Figur der Erde”, Pub. Königl. Preuss. Geodidät Inst., Berlin.
- [7] Campbell, L. and Moffat, J.W.. : 1983, *Astrophys. Jour.*, **275**, L77.
- [8] Chandrasekhar, S. : 1933, *Monthly Notices Royal Astron. Soc.*, **93**, 390.
- [9] Christensen-Dalsgaard, J., Dappen, W. Antia, H.M. : 1996, *Science*, **272**, 1296.
- [10] Clairaut, A.C. : 1743, “Théorie de la Figure de la Terre”, in *Principes de l’Hydrostatique*, Durand ed., Paris.
- [11] Corbard, T. and Thompson, M.J. : 2002, *Solar Phys.*, **205**, 211.
- [12] Corbard, T., Dikpati, M., Gilman, P.A. and Thompson, M.J. : 2002, in A. Wilson (ed.), *The SOHO 11 Symposium on From Solar Min to Max : Half a Solar Cycle with SOHO*, 11-15/03/2002, Davos, Switzerland, ESA SP-508, Noordwijk : ESA Publications Division, ISBN 92-9092-818-2, p. 75.

- [13] Dikpati, M., Corbard, T., Thompson, M.J. and Gilman, P.A. : 2002, *Astrophys. Jour.*, **575**, L41.
- [14] Di Mauro, M. : 2003, in “*The Sun Surface and Subsurface*”, Lecture Notes in Physics, 599, Springer ed., p. 58.
- [15] Domiciano de Souza, A., Vakili, F., Jankov, S., Janot-Pacheco, E. and Abe, L. : 2001, *Astron. & Astrophys.*, **393**, 345.
- [16] Godier, S. and Rozelot, J.P. : 1999, *Astron. & Astrophys.*, **350**, 310.
- [17] Godier, S. and Rozelot, J.P. : 2001, *Solar Phys.*, **199**, 217.
- [18] Goldreich, P. and Schubert, G. : 1968, *Astrophys. Jour.*, **154**, 1005.
- [19] Gough, D.O. : 1982, *Nature*, **298**, 334.
- [20] Hill, H.A. Bos, R.J. and Goode, P.R. : 1982, *Phys. Rev. Letters*, **49**, 1794.
- [21] Javaraiah, J. and Gokhale, M.H., *Sol. Phys.*, 1995, 173.
- [22] Javaraiah, J. and Rozelot, J-P. : 2004, *in preparation*.
- [23] Kosovichev, A.G. : 1996, *Astrophys. Jour.*, **469**, 61.
- [24] Kuhn, J.R., Bush, R., Scheick, X. and Scherrer, P. : 1998, *Nature*, 392, 155.
- [25] Landgraf, W. : 1992, *Solar Phys.*, **14**, 403.
- [26] Lefebvre, S. : 2003, Ph.D. Thesis, University of Nice-Sophia-Antipolis, 229 p.
- [27] Lefebvre, S. and Rozelot, J.P. : 2004, *Astron. & Astrophys.*, in press.
- [28] Lieske, J.H. and Null, G.W. : 1969, *Astron. J.*, **74** (2), 297.
- [29] Mecheri, R., Abdelatif, T. et al. : 2004, to be published.
- [30] Maeder, A. : 1999, *Astron. & Astrophys.*, **347**, 186.
- [31] McCarthy, D.D. : 2003, IERS Conventions (2003), IERS Technical Note 32, chap. 6 (available also on <http://maia.usno.navy.mil/conv2003.html>).
- [32] Milne, E.A. : 1923, *Monthly Notices Royal Astron. Soc.*, **83**, 119.
- [33] Modolensky, M.S. : 1988, *Geodezika i Kartografiya*, 5, 11.
- [34] Morel, P. : 1997, *Astron. & Astrophys.*, **124**, 597.
- [35] Moritz, H. : 1990, “*The Figure of the Earth*”, Wichmann ed.
- [36] Paternò, L., Sofia, S. and Di Mauro, M.P. : 1996, *Astron. & Astrophys.*, **314**, 940.
- [37] Pijpers, F. P. : 1998, *Monthly Notices Royal Astron. Soc.*, **297**, 76.
- [38] Pireaux, S. and Rozelot, J.P. : 2003, *Astron. & Space Science*, 284, 1159 (available also on [www : astro-ph/0109032](http://www.astro-ph/0109032)).
- [39] Radau, R. : 1885, “*Remarques sur la Théorie de la Figure de la Terre*”, *Bull. Astron.*, **2**, 157.
- [40] Richard, O. : 1999, Ph. D. Thesis, Université Pierre et Marie Curie (Paris 6), 149 p.
- [41] Roxburgh, I.W. : 2001, *Astron. & Astrophys.*, **377**, 688.

- [42] Rozelot, J.P. and Bois, E. : 1998, 19th National Solar Observatory Sacramento Peak Summer Workshop, New Mexico, Sept. 1997, “*New results concerning the Solar oblateness*”, ed. Balasubramaniam, K.S. , Harvey, J. and Rabin, D., published in Synoptic Solar Physics, J.A.S.P. Conference Series, Vol. 140, p. 75.
- [43] Rozelot, J.P. and Godier, S. : 2002, COSPAR, 16-23/07/2000, Warsaw, Poland, “*Current and future high resolution in-situ and remote sensing Solar physics missions*”, ed. Marsch, E. and Damé, L., published in Advances in Space Research, Vol. 29/12, p. 2089.
- [44] Rozelot, J.P., Godier, S. and Lefebvre, S. 2001, *Solar Phys.*, **198**, 223.
- [45] Rozelot, J.P and Lefebvre, S. 2003, in “*The Sun Surface and Subsurface*”, Lecture Notes in Physics, 599, Springer ed., p. 4.
- [46] Rozelot, J.P., Lefebvre, S. and Desnoux, V. : 2003, *Solar Phys.*, **217**, 39.
- [47] Ulrich, R. K. and Hawkins, G. W. : 1981a, *Astrophys. Jour.*, **246**, 985 (and erratum, 1981b, *Astrophys. Jour.*, **249**, 831).
- [48] Wavre, R. : 1932, in “*Figures planétaires et géodésie*”, Gauthiers-Villars ed., Paris.
- [49] Yuan, D-N., Sjogren, W.L., Konopliv, A.S., Kucinskas, A.B. : 2002, *J. Geophys. Res. E*, **106**, 23377.
- [50] Williams, J.G., Boggs, D.H., Dickey, J.O. and Folkner, W.M. : 2001, 9th Marcel Grossmann meeting 2000, “*Lunar Laser tests of gravitational physics*”, To be published in World Scientific.

References	Method	J_2	J_4	J_6	Others
Ulrich & Hawkins (1981)	SSE + spots rotation law	$(10-15) 10^{-8}$	$(0.2-0.5) 10^{-8}$		
Gough (1982)	First determination of helioseismic rot. rates	$36 10^{-7}$			
Campbell & Moffat (1983)	Planetary orbits	$(5.5 \pm 1.3) 10^{-6}$			
Landgraf (1992)	Astrometry of minor planets	$(0.6 \pm 5.8) 10^{-6}$			
Lydon & Sofia (1996)	SDS Experiment	$1.84 10^{-7}$	$9.83 10^{-7}$	$4 10^{-8}$	$J_8 = -4 10^{-9}$ $J_{10} = -2 10^{-10}$
Paternò <i>et al.</i> (1996)	SSE + empirical rotation law and SDS	$2.22 10^{-7}$			
Pijpers (1998)	SSE + GONG and SOI/MIDI data Weighted value	$(2.14 \pm 0.09) 10^{-7}$ $(2.23 \pm 0.09) 10^{-7}$ $(2.18 \pm 0.06) 10^{-7}$			
Armstrong & Kuhn (1999)	Vect. Spher. Harm. numerical error	$-0.222 10^{-6}$ $0.002 10^{-6}$	$3.84 10^{-9}$ $0.4 10^{-9}$		
Godier & Rozelot (1999)	SSE + Kosovichev law	$1.6 10^{-7}$			
Roxburgh (2001)	SSE + 2 models of rotation law	$2.208 10^{-7}$ $2.206 10^{-7}$	$-4.46 10^{-9}$ $-4.44 10^{-9}$	$-2.80 10^{-10}$ $-2.79 10^{-10}$	$J_8 = 1.49 10^{-11}$ $J_8 = 1.48 10^{-11}$
Rozelot <i>et al.</i> (2001)	Theory of Figures	$-(6.13 \pm 2.52) 10^{-7}$ Note 3	$3.4 10^{-7}$ Note 4		
Rozelot & Lefebvre (2003)	Theory of Figures	$-6.52 10^{-7}$	$4.20 10^{-7}$	$-9.46 10^{-9}$	$J_8 = 2.94 10^{-13}$
Present paper	SSE + Subsurface gradient of rotation (SGR)	$-2.28 10^{-7} \pm 15 \%$	very sensitive to SGR range : $\pm 20\%$		

Note 1 : SDS stands for Solar Disk Sextant

Note 2 : SSE stands for Stellar Structure Equations (based on Eqs. 4 and 6).

Note 3 : The apparent large error comes from the fact that the value is a weighted average of several rotation rates.

Note 4 : A mistake has been made in the second term of J_4 (i.e. mA_4), which was incorrectly multiplied by “ f ” in the computations.

TAB. 1 – Solar gravitational multipole moments quoted from different authors and methods.